Satisfying the demand in a timely fashion is a critical task in any company. In a service sector like telecommunications where sold information transmission units cannot be stored in inventory or cannot be backordered, having appropriate capacity to fulfill all demand becomes crucial. In order to achieve this goal, not only a well designed planning system but a sound capacity expansion strategy is necessary. In this paper, capacity planning in a telecommunications network is studied through the novel application of inventory control techniques aiming to meet the demand to a certain service level. In addition, a capacity expansion plan via equipment purchase is carried out through a mathematical programming model. The use of inventory control techniques in the telecommunications industry is the principal contribution of this work.

**Significance:** The model is implemented and tested by one of the major telecommunications provider in Mexico. The results indicate that the efficiency achieved through the integration of such techniques can become highly attractive for further applications in the industry.

**Keywords:** Inventory Control, Telecommunications, Optimization, Capacity Planning.

(Received ; Accepted )

1. INTRODUCTION

A telecommunications system typically consists of a physical infrastructure through which information is transported from a source to a destination. Among the most important types, information comprises data and voice (Veerasamy and Venkatesant, 1999). A telecommunications network is generally formed by a series of workstations, coordinated by servers and a variable group of autonomous devices, such as routers and switches. An illustrative example of a telecommunications network can be seen in Figure 1. Each active device that intervenes in the communication in an autonomous fashion is called ‘a node’. All nodes communicate among each other directly through information transportation networks.

![Telecommunications Network Based on Client-Server Architecture.](image)
The communication systems in network are traditionally based on the client-server architecture (Sprangins et al., 1992) where ‘the client’ is the computer that sends a petition or requests a service, while ‘the server’ is the remote computer that controls this service. Each requested client service is measured by ‘bandwidth units’. Therefore, a telecommunications network should have enough available capacity in terms of bandwidth units in their nodes in order to execute all client services.

In telecommunications, when a service contract is signed, the service provider guarantees to satisfy at least certain percentage of bandwidth requirements of the customer during the contracted period. Therefore, a decision maker in the service-providing entity has the task of determining the time at which capacity must be expanded, the amount of bandwidth necessary for each expansion, as well as the appropriate equipment that will provide the capacity to satisfy the committed service level. In other words, the decision maker faces a capacity planning problem.

Several factors complicate a capacity planning problem in telecommunications. Saniee (1995) states that the difficulty of each problem depends on the transmission technology or the combination of the used technologies to satisfy the demand of each one of the nodes for each period in the planning horizon.

One of the most important aspects that should be considered to plan capacity in a telecommunications network is the survival. The survival is defined as the ability of reestablishing the net services in catastrophic events or failures (Melian et al., 2003). One of the catastrophic but quite common failures is to have a wrong node or a wrong demand forecast for a node. This type of failures is difficult to capture mathematically, however, due to its highly stochastic nature.

On the other side, each telecommunications service has different characteristics that require different transport networks. Therefore, each problem has different constraints and objectives which can be solved with a great number of methodologies. For example Premkumar and Chu (1999) propose a heuristic method based on genetic algorithms to design and develop the infrastructure of a telecommunications network. Chen and Chan (2001) suggest a heuristic method based on shortest-path and maximum-flow algorithms, to determine the necessary bandwidth flow in a similar network and Melian et al., (2002) developed a mixed integer mathematical program model to determine the amount of optic fiber capacity expansion.

In the telecommunications industry, after demand forecasts of clients becomes available, it is desirable to know whether the quantity of physical equipments on hand will be enough to supply required service to each client or more equipment will be needed. When forecasted demand requires more bandwidth than the capacity on hand, then the problem becomes when and how many new types of equipment to order to meet the excess demand. In manufacturing environment contexts, these questions define the inventory control policy of the company. Therefore, any well established methodology for inventory control can be applied to a telecommunications capacity planning problem. This, however, has not been tried previously according to our literature search.

The remainder of the paper is organized as follows: In the following section, we introduce the typical capacity planning problem and the notation used in the paper. Section 3 is devoted to determining the capacity levels using inventory theory approach for a telecommunications capacity planning problem. The equipment selection problem together with the formulation is explained in Section 4. We illustrate the solution technique with a case study and discuss the findings in Section 5. We conclude with final remarks in Section 6.

2. TYPICAL CAPACITY PLANNING PROBLEM

The problem that any telecommunications company faces is to determine the amount of bandwidth required to satisfy certain demand without incurring in excessive costs to maintain installed and potentially unused capacity. The important balance is here in maintaining an equipment inventory that guarantees a certain service level to the client.

Since demand data used for planning is usually a forecast, one of the common practices is to set a safety stock to protect against the possible fluctuations in the realized demand. In the related literature, another common approach to capacity planning in telecommunications is to model and solve an optimization problem (Economides, 1999). In such problem the number of equipment pieces needed in a certain time period is to be determined with the objective of minimizing the cost associated with the equipment while fulfilling certain demand levels. Since most of the cases are defined as combinatorial problems with binary and integer variables, the resulting optimization model can become very difficult to solve when analyzing a large number of periods (Coyle, 1998). Combinatorial optimization problems, besides computational time and powerful computers, require many times highly specialized software. These associated costs could be avoided if a more efficient technique was employed to approach the capacity planning problem. With this in mind, the proposed approach in this work, as explained in Section 3, is to use inventory control techniques to deliver an efficient solution to the capacity planning problem in this industry.
2.1 Notation

Throughout the paper we use the following parameters:

\( a_i \): Ordering cost for a unit of bandwidth in period \( i \),

\( b_i \): Holding cost for a unit of bandwidth maintained in inventory in period \( i \),

\( f_i \): Cost of generating a purchase order,

\( C_i \): Capacity of bandwidth in period \( i \), while \( c_0 \) is the initial bandwidth capacity

\( D_i \): Customer demand in period \( i \) and

\( M \): A large positive number.

Two decisions need to be made in every period: 1) If an equipment purchase order will be given and 2) the amount of bandwidth that will be ordered. The associated decision variables are the following:

\( x_i \): The quantity of bandwidth ordered for the \( i \)th period,

\( y_i \): A binary variable that takes the value of 1 if an equipment purchase order is generated in period \( i \), and 0 otherwise,

Moreover, we have the additional decision variable to keep track of the unutilized bandwidth level:

\( I_i \): The quantity of unutilized bandwidth in period \( i \)

2.2. Mathematical model

A typical model to plan the capacity for the next \( n \) periods with the objective of minimizing cost: will be as follows:

Minimize \( \sum_{i=1}^{n} (a_i x_i + b_i I_i + f_i y_i) \)

Subject to

\( C_1 = C_0 \) \hspace{1cm} \ldots \hspace{1cm} \ldots (1) \\
\( C_i + x_i = C_{i+1} \) \( i = 1, 2, \ldots, n \) \hspace{1cm} \ldots \hspace{1cm} \ldots (2) \\
\( D_i \leq C_i \) \( i = 2, 3, \ldots, n \) \hspace{1cm} \ldots \hspace{1cm} \ldots (3) \\
\( C_i - D_i = I_i \) \( i = 1, 2, \ldots, n \) \hspace{1cm} \ldots \hspace{1cm} \ldots (4) \\
\( x_i \leq My_i \) \( i = 1, 2, \ldots, n \) \hspace{1cm} \ldots \hspace{1cm} \ldots (5) \\
\( x_i \geq 0 \) \( i = 1, 2, \ldots, n \) \hspace{1cm} \ldots \hspace{1cm} \ldots (6) \\
\( y_i \in \{0,1\} \) \( i = 1, 2, \ldots, n \)

Where constraint (1) guarantees that the planning period will start with the initial capacity. Constraint set (2) ensures the balance of the bandwidth capacity in each period, respectively. In each period, capacity should be at least as much as the customer demand, which is captured by the constraint set (3). Also, in each period, the company incurs a cost for each unit of bandwidth that is not used. Constraint set (4) calculates the bandwidth quantity that is not used in any period. Finally, constraint set (5) guarantees that any extra bandwidth capacity is from the ordered equipment. Non-negativity constraints (6) are also included. The objective is to minimize the total cost of acquiring new equipment, additional bandwidths and cost of carrying unused capacity.

The idea in this study is to solve the problem of capacity planning for a telecommunications network, laid out here as a mixed integer programming problem (MIP), with a novel approach that makes use of inventory control techniques.

3 INVENTORY THEORY APPROACH FOR A TELECOMMUNICATIONS PROBLEM

The characteristics of the capacity planning problem in telecommunications make it very similar to an EOQ-type inventory control problem. The objective of the basic EOQ model (Harris, 1913) is to minimize the annual cost per inventory unit, which includes cost of generating an order (set-up cost) and interest and depreciation on stock (inventory holding cost) in addition to unit acquisition cost. Since there is a conflict between the set-up and the holding cost with respect to the size of the orders, the EOQ model identifies the optimum order quantity, which is fixed for fixed demand rate. In the original EOQ model the replenishment lead time -the time period between the time that the articles are ordered and the time that they become available- is assumed to be zero. Nevertheless, as long as the demand rate remains deterministic, introduction of a known non-zero lead time (L) presents no difficulty. Inventory level behavior in time stays unchanged. In the beginning of a period of time (T) the inventory level starts in (Q) articles, as the time passes the inventory level decreases with rate (D). When the inventory level hits a certain level an order is
placed. This point indicates the moment in which we have to re-order the inventory and will be denoted, as \((r)\). The requested order arrives exactly \(L\) time units later, just as the inventory hits zero (Silver et al., 1998). A typical inventory diagram is shown in Figure 2.

![Inventory Level Graph](image1)

Figure 2. Behavior of Inventory level with Time in EOQ Model.

In the telecommunications context, the main purpose is to offer service to all clients, increasing the capacity when required. The capacity in this context is the quantity of available bandwidth to give service to the clients. As it can be seen in Figure 3, company has an initial capacity \((C_0)\) with which the demand will be supplied, as time passes the demand increases and at reorder point \((r)\), \((Q)\) units of bandwidth are ordered, taking into account a lead time \((L)\).

![Capacity Level Graph](image2)

Figure 3. Behaviour of Capacity Level for a Telecommunications network.

The slack capacity of the company can be calculated as the mathematical difference between the quantity of bandwidth units that are available for the consumption (supply) and the quantity of bandwidth units that clients consume (demand). As shown in Figure 4, when we plot the behavior of the slack capacity in time, we obtain a graph very similar to the inventory diagram of EOQ-model. Company starts with an initial number of units of bandwidth and, as the time passes, the slack between demand and the capacity diminishes until arriving to a point \((r)\), which indicates the moment to reorder \(Q\) units of extra capacity, taking into account lead time \((L)\).
Such similarities between the behavior of inventory level with time in EOQ model (Figure 2) and behavior of slack capacity in telecommunication networks (Figure 4), justifies the use the fixed order quantity inventory control models to give solution to the capacity planning problem in telecommunications networks. The application of several fixed order quantity inventory models for the capacity planning problem was indeed explored in Álvarez Herrera and Cabrera Ríos, 2005. The evaluation of these models suggests that it is sensible and advisable to use a continuous review, order-point (r), fixed order quantity (Q), (Q, r)-model where safety stock values are determined based on a specified cycle service level. In the service level (Q, r) model, certain level of customer service is determined as the probability of no stockout per replenishment cycle under stochastic demand. Multiple case studies are demonstrated in Álvarez Herrera and Cabrera Ríos, 2006 for service level (Q, r) model with results that agree with the typically adequate performance of inventory models.

4. EQUIPMENT SELECTION PROBLEM

Capacity planning in the telecommunications industry not only implies the determination of the bandwidth amount that is necessary to acquire in order to satisfy the demand, it also includes the determination of the type and quantity of equipment that should be purchased to meet the demand. For this purpose, a more traditional mathematical programming problem was used as described next.

4.1. Description of the problem

A company of telecommunications needs to buy routers that will provide ports (in units of bandwidth) for the next operation of the network. There are two or more alternatives. Each router has a shelf, which has a capacity in number of cards, and these cards in turn contain a maximum number of ports. Between two cards, a separator of card is needed. The cost structures, the routers capacities and the characteristics of each router’s accessories are different, and the objective is to select the best option with minimum cost to satisfy the demand.

4.2. Notation

In order to make the appropriate equipment selection -in this context selection of routers- an integer programming model is used. The integer programming model determines the number and type of routers that should be acquired in order to minimize the total acquisition cost. To depict this problem we introduce additional notation:

\[ r_i \]: The unit cost of router type \( i \),
\[ c_i \]: The unit cost of a card used in the router type \( i \),
\[ k_i \]: The unit cost of a card separator used in the router type \( i \),
\[ t_i \]: The maximum capacity of cards in router,
\[ p_i \]: The maximum capacity of ports for a card used in the router type \( i \),
\[ s_i \]: The maximum capacity of cards for separator used in the router type \( i \),
\[ D \]: the demand in number of ports.

Moreover, the associated decision variables are the following:
The number of router type $i$,

The number of card separators for the router type $i$,

The number of cards for the router type $i$.

### 4.3. Mathematical Model

The integer programming model used is as follows:

$$\text{Minimize } \sum_{i=1}^{n} (r_i x_i + c_i z_i + k_i w_i)$$

Subject To

$$\sum_i p_i z_i \geq D \quad \text{...} \quad \text{....}(7)$$

$$t_i x_i \geq z_i \quad \forall i = 1,2,...,n \quad \text{...} \quad \text{....}(8)$$

$$s_i w_i \geq z_i \quad \forall i = 1,2,...,n \quad \text{...} \quad \text{....}(9)$$

$$x_i, z_i, w_i \in Z^+ \quad i = 1,2,...,n \quad \text{...} \quad \text{....}(10)$$

where equation (7) guarantees that total ports purchased is greater than the demand, while equation sets (8) and (9) ensure that, for each router the number of cards acquired is less than the total maximum card capacity for routers and card separators, respectively. Non-negativity constraints (10) are also included. The objective is to minimize the total cost of acquiring new routers and their necessary components: cards and card separators.

### 4.4. Decision Support System

Capacity planning comprises a series of strategic decisions that fixes the course of action for the future tactical and operational decisions, establishing principles and constraining further decisions by determining time and quantities. A bad capacity planning followed by a bad equipment acquisition can bring significant economic damages to the company. A user-friendly decision support system (DSS) to aid planning and equipment selection is posed to have a positive impact. With this idea in mind, a DSS was developed in MS Excel to accommodate both the inventory control-based solution to the capacity planning problem and the mathematical programming model for the equipment purchasing problem.

The DSS requires $n$ monthly demand forecasts as well as the unit bandwidth cost, initial capacity level and service level requirement as input parameters. After solving the decision making problem using the modeling approaches explained in sections 3 and 4.1, the system gives the size and the time of capacity expansion (in bandwidth units), as well as the equipment selection that will minimize the cost while satisfying a particular service level. The use of the DSS is demonstrated here through a case study.

### 5. CASE STUDY

The DSS described previously was implemented using historical data of one of the largest telecommunications company in Mexico. The company offers a wide portfolio of telecommunications services to different sized companies from all sectors. It also offers services comprising voice and data transmission to virtual private networks (VPNs), integrated telecommunications packages and managed services. Their demand level changes quite often, and so do their capacity requirements. In this section, historical capacity and demand data for a representative year is analyzed. The initial capacity of the network is 128 units of bandwidth and the demand is tabulated and visualised in Figure 5.
In our analysis the constant lead time used by the company is reported to be 4 months for their suppliers and 3 types of equipments are available to provide the capacity. The characteristics of each equipment are summarized in Table 1.

<table>
<thead>
<tr>
<th>Equipment type</th>
<th>Card Capacity per Equipment</th>
<th>Bandwidth Capacity per card</th>
<th>Unit Cost per equipment</th>
<th>Unit Cost per card</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>5</td>
<td>18</td>
<td>$1,000.00</td>
<td>$1,500.00</td>
</tr>
<tr>
<td>II</td>
<td>10</td>
<td>6</td>
<td>$1,500.00</td>
<td>$1,000.00</td>
</tr>
<tr>
<td>III</td>
<td>8</td>
<td>12</td>
<td>$1,500.00</td>
<td>$900.00</td>
</tr>
</tbody>
</table>

Table 1. Equipment characteristics for the case study

When we solved this capacity expansion problem with the DSS at a service level of 85%, the results suggested that two capacity expansions be carried out: the first one in month 5 and the second one in month 10. The first expansion requires the purchase of 75 units of bandwidth via equipment type III with 7 cards. For the second expansion of 31 units of bandwidth, equipment type I with 2 cards should be bought, incurring cost of $11,800.00 in total. Figure 7 shows us the results obtained by using the DSS for the capacity expansion schedule.

A closer look to the solution prescribed by the DSS, will show how the fact that it takes 4 months from the time when a decision is made to expand the network’s capacity to actually having that additional capacity leads to potentially not meeting the demand on months 2 thru 4. Increasing the service level will certainly not improve the solution here, as it has to do with the suppliers’ lead time.

Because varying any of the parameters in the DSS will show the results for the capacity expansion schedule in a matter of seconds, using inventory control techniques in this phase of the planning problem dramatically decreases the required analysis time as opposed to solving a mixed integer programming problem. This is specially true considering that many times different demand forecasts must be evaluated to decide if an investment will be made in a capacity expansion.

As a final comment, we must add that the distribution of the DSS throughout the company was extremely easy with the chosen platform. It also took very little effort to the users to become acquainted with the tool.
### Capacity Expansion

<table>
<thead>
<tr>
<th>Month</th>
<th>Demand (bandwidth units)</th>
<th>Capacity Expansion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Place order in Month</td>
<td>Installed Capacity</td>
</tr>
<tr>
<td>1</td>
<td>115</td>
<td>128</td>
</tr>
<tr>
<td>2</td>
<td>123</td>
<td>128</td>
</tr>
<tr>
<td>3</td>
<td>126</td>
<td>128</td>
</tr>
<tr>
<td>4</td>
<td>127</td>
<td>128</td>
</tr>
<tr>
<td>5</td>
<td>136, 75, 1</td>
<td>203</td>
</tr>
<tr>
<td>6</td>
<td>146</td>
<td>203</td>
</tr>
<tr>
<td>7</td>
<td>161</td>
<td>203</td>
</tr>
<tr>
<td>8</td>
<td>161</td>
<td>203</td>
</tr>
<tr>
<td>9</td>
<td>165</td>
<td>203</td>
</tr>
<tr>
<td>10</td>
<td>165, 31, 6</td>
<td>234</td>
</tr>
<tr>
<td>11</td>
<td>188</td>
<td>234</td>
</tr>
<tr>
<td>12</td>
<td>191</td>
<td>234</td>
</tr>
<tr>
<td>TOTAL ACQUISITION:</td>
<td></td>
<td>106</td>
</tr>
</tbody>
</table>

Figure 7. Results for the capacity expansion schedule obtained through the DSS

### 6. CONCLUSIONS

In this work, inventory control techniques were applied in a real capacity planning problem in telecommunications, capitalizing in the similarities between the two problems. The problem of inventory control has been largely studied, however to the best extent of our knowledge, no real life application to capacity planning has been reported in the telecommunications area. Using inventory control models has been shown to give an efficient solution to the original capacity planning problem. The associated equipment selection problem was approached through the use of a typical mathematical programming model. Finally a DSS was developed to integrate the proposed techniques for their actual application in the industry. The results presented in this work underline the importance of establishing similarities among problems of different areas to capitalize in their corresponding solution techniques. In particular, the use of standard industrial engineering techniques in new areas, such as telecommunications, does provide significant development opportunities for the discipline.

### 7. REFERENCES


8. ACKNOWLEDGEMENTS

The authors are grateful for the scholarship support to Mr. Álvarez from CONACYT (National Mexican Concil for Science and Technology), as well as to the people from the company, who prefer to remain anonymous, for their continuous support for this project.