An integrated model for the frequency and timetabling problem

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Abstract

The process of urban public transportation planning commonly include four basic activities usually executed in sequence: Network design, Timetabling, Vehicle scheduling and Crew scheduling. In this work we present an integrated model for the minimum frequency and timetabling. We use two metaheuristics to solve it. The main scientific contribution of this paper is the development of an integrated MILP model to construct timetable by selecting frequencies in such a way that multiple objectives are optimized, like operational cost, synchronization, waiting time and smooth transitions between periods. Finally we show and analyze some experimental results.

Keywords: frequency; timetabling; multiobjective; multiperiod.
1 Introduction

In this paper is addressed the problem urban bus planning. This is a process which is usually divided into four phases: network design, timetabling constructing, vehicle scheduling, and crew scheduling. Frequently, these phases are executed sequentially. In this paper we are tackling the bus timetable construction problem of an urban bus transport network, which is accomplished usually in two steps: first for each scenario (covering a concrete planning period) bus frequencies are calculated for each route in the network, then bus departures are settled for each route in the network, were previously calculated frequencies are employed as a lower bound for the total departures programmed in each route in each scenario. This approach have as a main drawback that often frequencies should be adjusted for getting acceptable timetables for planners.

The main scientific contribution of this work is the development of an integrated multi-objective mathematical model to construct multi-period urban bus timetables, which also allows for smooth transitions between adjacent planning periods.

Recently Ibarra-Rojas & Ríos-Solís [8] have shown formally that the timetabling problem is NP-Hard, so we implemented two multiobjective metaheuristics for exploring the effectiveness of the proposed model when it is faced with simulated real instances of the problem.

2 Problem Description

The planning process is divided into four phases: network design, timetabling, vehicle scheduling and crew scheduling [4]. The timetabling phase has two activities frequency determination and timetable construction, these activities are executed sequentially.

The problem addressed in this work is about to develop a mathematical model to calculate in an integrated way the frequencies and the timetables for the operation of the urban transport. Considering multiple periods with different demand and getting a smooth transition between them. That is from a period with high demand to a period with low demand or the other way.

Additionally, we are taking into account multiple objectives derived from the requirements of the social actors involved in the process, like synchronization between bus routes in a specific node, cost, waiting time and a minimum difference with a smooth transition timetable.

The main challenge being faced here is to incorporate objectives that represent interests from all social actors involved in the process. Another challenge is to develop the planning for different periods with different demand. As the problem have been shown to be NP-Hard, so another challenge in this work is to develop efficient algorithms to the solutions of it.

3 Related Work

Through the literature we can find that the timetabling problem have been tackled from different forms, one of them is the utilization of exact methods such as case Ceder [3] he creates a timetable with maximal synchronization. Also Eranki [6] proposes a model to create timetables with maximal synchronization using time windows, she uses a heuristic to solve the problem, but she did not
Consider more criteria.

There are other investigations about timetables that use constraint programming [1] where the author presents a model considering different characteristics of the transport system (passenger requirements, budget constraints, level of service) and they solve it with constraint programming.

Exist some other investigations of timetables where the authors use heuristics like GRASP, we can mention Mauttone & Urquhart [10] who develop an heuristic based on GRASP optimizing simultaneously different objectives of passengers and schedulers.

Trough the literature there are approaches, combining two phases of the urban transport process such as Szeto & Wu [12], they propose an integrated solution for the bus network design and frequency setting problems simultaneously using a genetic algorithm, which tackles the route network design problem is hybridized with a neighborhood search heuristic, which tackles the frequency setting problem; there are also approaches solving two phase sequentially, like the research made by Chakroborty [5], who combines the transit routing and scheduling phases using a genetic algorithm, in his approach he tries to minimize the transfer time and the waiting time; another research that combines several phases is the one proposed by Zhao & Zeng [13] they present a metaheuristic method for optimizing transit networks, including route network design, vehicle headway and timetable assignment, the goal is to identify a transit network that minimizes a passenger cost function; their metaheuristic combines simulated annealing, tabu and greedy search methods.

Although there are investigations about timetable that tackled the problem considering different criteria, visualize the problem like a multiobjective, approaches that combines (sequentially or integrated) different phases of the transport system, even exists investigations that talk about the smooth transition between periods, such as Ceder [4], he proposes two techniques to handle the smooth transitions between periods with different demand; even headways with smooth transitions and headways with even average loads. There is no article in the literature checked that integrates simultaneously the minimum frequency problem and the timetabling problem considering multiple objectives.

4 Proposed Heuristic

4.1 Assumptions.

Here we present the assumptions of the problem addressed in this work:

- Demand does not change significantly in each period and it is known in advanced.
- Average travel time from each route in each period is known.
- Periods length must be enough to allow the schedule the needed departures.
- The planning requirements must ensure the satisfaction of the demand during the planning period established.
- We are only taking into account those ships within the same period.
4.2 Mathematical model.

Sets

- $M$: Set of routes.
- $K$: Set of nodes.
- $V$: Set of periods.
- $B_{ij}^v$: Set of pairs of nodes where potentially synchronize the routes $i$ and $j$.
- $J(i)$: Set of routes which have common nodes with the route $i$.

Variables

- $X_{ip}^v = 1$ there is a trip in the route $i$ with departure time in the interval $(p \cdot H_{min}^v, p + 1 \cdot H_{min}^v + g)$ in the period $v$ and $0$ otherwise.
- $\alpha_{ip}^v \in (p \cdot H_{min}^v, p + 1 \cdot H_{min}^v + g)$ if $X_{ip}^v = 1$ and $0$ if $X_{ip}^v = 0$.
- $Y_{ijkupq}^v = 1$ if the bus of the route $i$ with departure time in the interval $(p \cdot H_{min}^v, p + 1 \cdot H_{min}^v + g)$ and the bus of the route $j$ with departure time in the interval $(q \cdot H_{min}^v, q + 1 \cdot H_{min}^v + g)$ arrive to the segment $k$-$u$ (fixed synchronization node) within the window time and $0$ otherwise.
- $\mu_{ip}^v$: Represents the absolut difference in relation to the closer departure time of the even average loads method if there is a trip in the route $i$ in the interval $(p \cdot H_{min}^v, p + 1 \cdot H_{min}^v + g)$ in the period $v$.
- $Z_{ijk}^v$: the difference between the arrival time of the routes $i$ and $j$ in the segment $k$-$u$ in the period $v$.

Parameters

- $G^v$: Number of trips in the period if we use a frequency equal to $H_{max}^v$.
- $P_{max}^v$: Maximum load of passengers in the route $i$ in the period $v$.
- $P_{maxd}^i$: Maximum load of passengers on bord in the route $i$.
- $d_{pi}^v$: Desired occupancy of the bus in the route $i$ in the period $v$.
- $P_{as}^i$: Total passengers/km in the route $i$ in the period $v$.
- $L_i$: Length of the route $i$.
- $cap_i^v$: Bus capacity of the route $i$ in the period $v$.
- $l_k$: Length of the segment $k$.
- $\beta_i^v$: Percentage allowed of the route $i$ of exceed the load in the period $v$.
- $H_{max}^v$: Minimum headway of the route $i$ in the period $v$.
- $H_{min}^v$: Maximum headway of the route $i$ in the period $v$.
- $T_i^v$: Planning period; $[T_{ini}^v, T_{fin}^v]$.
- $T_{ini}^v$: Beginning time of the planning period $v$.
- $T_{fin}^v$: Ending time of the planning period $v$.
- $\gamma_i^v$: Desired time before the end of the period $T_i^v$ for the last departure of the route $i$ in the period $v$.
- $W_{max}^v$: Maximum window time for the route $i$ in the period $v$.
- $W_{min}^v$: Minimum window time for the route $i$ in the period $v$.
- $t_{ik}^v$: Travel time from the origen point of the route $i$ to the segment $k$ in the period $v$.
- $\delta_{ijk}^v$: Minimum time the passenger needs to change from segment $k$ of the route $i$ to the segment $u$ of the route $j$ in the period $v$.
- $\pi_{ijk}^v$: Number of passengers changing from segment $k$ of route $i$ to the segment $u$ of the route $j$.
- $P_{max}^v$: Maximum load average of passengers on bord of route $i$ in the period $v$.
- $MC_i^v$: Method applied to determine the frequency in the period $v$. 

3
\( f_{mv}^i \): Minimum frequency required to satisfy the demand of the route \( i \) in the period \( v \).
\[ f_{mr}^i = \frac{P_{\text{max}}^v}{d_i^v}. \]

\( C_{ip}^v \): Timetable calculated with the even average load method. 1 if there is a departure in the instant \( p \) for the route \( i \) in the period \( v \).

\( \text{CostoFijo}_i^v \): Fixed cost for the route \( i \) in the period \( v \).

\( \text{CostoVariable}_i^v \): Variable cost for the route \( i \) in the period \( v \).

\( P_k^v \): Average of passengers on board in the segment \( k \) in the period \( v \).

\( s_{jk}^v \): Holding time of the route \( j \) in the segment \( k \) during the period \( v \).

\[
\begin{align*}
\text{min} & \sum_{i \in M} \sum_{v \in V} (\text{CostoFijo}_i^v + \text{CostoVariable}_i^v \cdot L_i \cdot \sum_{p \in N_v} X_{v ip}) \quad (1) \\
\max & \sum_{i \in M} \sum_{j \in J(i)} \sum_{(k,u) \in B_{ij}^v} \sum_{v \in V} \sum_{p \in N_v} \sum_{q \in N_v} Y_{v ikupq}^v \quad (2) \\
\text{min} & \sum_{i \in M} \sum_{j \in J(i)} \sum_{v \in V} \sum_{(k,u) \in B_{ij}^v} \pi_{v ikju}^v \cdot Z_{v ikju}^v \quad (3) \\
\text{min} & \sum_{v \in V} \sum_{i \in M} \sum_{p \in N_v} \mu_{v ip}^v \quad (4)
\end{align*}
\]

s.t.

\[
\sum_{p \in N_v} X_{v ip} \geq \frac{P_{\text{max},d_i^v}}{d_i^v}; v \in V; i \in M \quad (5)
\]

\[
\sum_{p \in N_v} X_{v ip} \geq \frac{P_{\text{max},d_i^v}}{d_i^v}; v \in V; i \in M \quad (6)
\]

\[
\sum_{p \in N_v} X_{v ip} \geq \frac{P_{\text{max},d_i^v}}{d_i^v} \cdot L_i \quad \& \& \\
\sum_{p \in N_v} X_{v ip} \geq \frac{P_{\text{max},d_i^v}}{\text{cap}^v_i} \quad (7)
\]
\[ MC^v = 4 \Rightarrow \sum_{p \in N^v} X_{ip}^v \geq \frac{P_{as_i}^v}{d_i^v \cdot L_i} \land \land \]

\[ \sum_{p \in N^v} X_{ip}^v \geq \frac{P_{max_i}^v}{cap_i^v} \land \land \]

\[ \sum_{k \in I^v} l_k \leq \beta_i^v \cdot L_i; \]

\[ v \in V; i \in M; I^v = \{ k \mid \frac{P_k^v}{f_{mr_i}^v} \geq d_i^v \} \]  

(8)

\[ X_{ip}^v = 0 \iff \alpha_{ip}^v = 0 \]  

\[ \forall v \in V, \forall i \in M, \forall p \in N^v \]  

(9)

\[ \sum_{p \in N^v} X_{ip}^v \geq \max\{|G_r^v|, f_{mr_i}^v\}; \]  

\[ v \in V; i \in M \]  

(10)

\[ \alpha_{ip}^v \leq \alpha_{il}^v \land \alpha_{il}^v > 0 \Rightarrow \alpha_{ip}^v \leq H_{max_i}^v \]  

\[ \forall v \in V, \forall i \in M, \forall p, l \in N^v \]  

(11)

\[ \alpha_{ih}^v > 0 \land \alpha_{ip}^v > 0 \land \alpha_{ip}^v = \min(\alpha_{il}^v) \Rightarrow \]  

\[ \alpha_{ih}^v + H_{min_i}^v \leq \alpha_{ip}^v \leq \alpha_{ih}^v + H_{max_i}^v \]  

\[ \forall v \in V, \forall i \in M, \forall p, h, l \in N^v \land l > h \]  

(12)

\[ \alpha_{ip}^v > 0 \land \alpha_{ip}^v = \max(\alpha_{il}^v) \Rightarrow \]  

\[ T_{fin}^v - \gamma_i^v \leq \alpha_{ip}^v; \]  

\[ \forall v \in V, \forall i \in M, \forall p, l \in N^v \]  

(13)

\[ Y_{ijkupq}^v = 1 \iff \]  

\[ \alpha_{ip}^v > 0 \land \alpha_{jq}^v > 0 \land W_{min_i}^v - t_{ju}^v - s_{jk}^v + \alpha_{ip}^v + t_{ik}^v + \delta_{ijk}^v \leq \alpha_{jq}^v \land \]  

\[ \alpha_{jq}^v \leq W_{max_i}^v - t_{ju}^v - s_{jk}^v + \alpha_{ip}^v + t_{ik}^v + \delta_{ijku}^v \land \alpha_{jq}^v + t_{ju}^v + s_{jk}^v \geq \alpha_{ip}^v + t_{ik}^v + \delta_{ijk}^v; \]  

\[ v, v^* \in V; i \in M; (k, u) \in B_{ij}^v; \]  

\[ j \in J(i); p \in N^v, q \in N^{v*}, v^* \leq v \]  

(14)
\[ Y_{ijkupq}^v = 1 \Rightarrow Z_{ijku}^v = \max(\alpha_{jq}^v + t_{ju}^v + s_{ju}^v) - (\alpha_{ip}^v + t_{ik}^v + \delta_{ijk}^v) \]

\[ v, v^* \in V; \quad i \in M; \quad (k, u) \in B_{ij}^v \]

\[ j \in J(i); \quad p \in N_v^i, \quad q \in N_{v^*}, \quad v^* \leq v \]

(15)

\[ Y_{ijkupq}^v = 0 \Rightarrow Z_{ijku}^v = 0; \]

\[ v \in V; \quad i \in M; \quad (k, u) \in B_{ij}^v \]

\[ j \in J(i); \quad p, q \in N_v \]

(16)

\[ \alpha_{ip}^v > 0 \Rightarrow \mu_{ip}^v = \min\{\text{abs}(C_{is}^{v^*} - \alpha_{ip}^v)\}; \]

\[ \forall v, v^* \in V, \forall i \in M, \forall p, s \in N_v, v^* \leq v \]

(17)

\[ \alpha_{ip}^v = 0 \Rightarrow \mu_{ip}^v = 0; \]

\[ \forall v \in V, \forall i \in M, \forall p \in N_v \]

(18)

\[ X_{ip}^v \in \{0, 1\}, \quad \alpha_{ip}^v \in \{T_{ini}^v, T_{fin}^v\}, \quad Y_{ijkupq}^v \in \{0, 1\}, \quad Z_{ijku}^v \in \mathbb{R} \]

\[ \mu_{ip}^v \in \mathbb{R} \quad \forall i, j \in M, \forall k, u \in K, \forall v \in V, \forall p, q \in T^v \]

The model consists of 4 objective functions, the first objective function (1) minimize the total cost. We are considering a fixed and a variable cost which it is affected by the long of the route and the quantity of ships made by the route in the period. The second function (2) is to maximize the number of synchronizations between two bus routes in a period. The third function (3) is to minimize the transfer times and the fourth function (4) is to minimize a penalty for do not accomplish the departure time obtained with a average loads method [4], which warranties a good transition between periods with different demand.

These objective functions are subjected to constraints of frequency (5-8) which indicate the minimum quantity of units that a route needs to satisfy the demand. The method I (5) represents the maximum load point during the day3, the method II (6) is based on the maximum load point during the period, method III (7) warranties the segment with maximum load will not present overcrowding and the method IV (8) sets a service level restricting a percentage of the total length of the route with overcrowding.

With constraint (9) we say if there is not a travel in the period v for the route i in the segment p then we do not assign a departure time. In constraint (10) the quantity of ships must be the maximum between the number of ships determined by the maximum headway and the minimum frequency, which satisfies the maximum load point, in this way we are warranting accomplish of the demand.

Constraint (11) indicates that the departure time of the first ship must be less or equal to the maximum headway. Constraint (12) is for the consecutive ships, here indicates that the departure
time must be between the minimum and maximum headway. For the last ship (13) we say the departure time must be between the end of the period and desired time.

The constraint (14) represents the synchronization which indicates when two bus of different routes arrives to a synchronization node between a window time and taking into account the transfer times, the permanence time in a node and the travel time, then there is a synchronization. The constraints (15) and (16) counts the time the passengers wait to do the transfer.

The constraints (17) and (18) are related to objective function (4) these represent the difference between the ship assign by our model and the closer ship according the method of average loads [4].

5 Decision support methodology.

In Figure 1 we show the first three phases in the decision making process[9], indicating the proposal made in each phase for the problem exposed in this work. It should be noted, in this approach the implementation phase is not addressed here.

The intelligence phase is covered with the mathematical model presented in the previous section, the design phase covers the optimization, in this case metaheuristics optimization (MOTS [7] and SSPMO [11]). Finally, in the selection phase we employ Promethee [2] because the generated ranking of alternatives offered allows the schedulers to choose the most attractive alternatives with regard to his own preferences.

Figure 1: Phases according Simon French [9].

<table>
<thead>
<tr>
<th>Phases</th>
<th>Actions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intelligence</td>
<td>Mathematical model</td>
</tr>
<tr>
<td>Design</td>
<td>Multiobjective optimization</td>
</tr>
<tr>
<td>Selection</td>
<td>MCDA</td>
</tr>
</tbody>
</table>

6 Computational experiments

Random instances were created and classified according quantity of periods, bus stops and routes in small, medium and large. In relation to synchronization, the instances were classified by density according to a percentage of combinations of bus stops in each route, see Table 1. The instances generator was developed in OPL.

Table 1: Characteristics of instances.

<table>
<thead>
<tr>
<th></th>
<th>Small</th>
<th>Medium</th>
<th>Large</th>
</tr>
</thead>
<tbody>
<tr>
<td>Periods</td>
<td>3-5</td>
<td>8-10</td>
<td>8-10</td>
</tr>
<tr>
<td>Nodes</td>
<td>10-18</td>
<td>19-23</td>
<td>35-50</td>
</tr>
<tr>
<td>Routes</td>
<td>2-4</td>
<td>5-8</td>
<td>8</td>
</tr>
<tr>
<td>Density</td>
<td>1%-2%</td>
<td>2%-4%</td>
<td>4%-7%</td>
</tr>
</tbody>
</table>
Of the 25 instances with we had made tests, we have selected three of them to show results and compare both algorithms, because in the other instances the behavior is similar. The data presented in Table 2 represents the data of the instances.

<table>
<thead>
<tr>
<th></th>
<th>Small</th>
<th>Medium</th>
<th>Large</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Routes</strong></td>
<td>3</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td><strong>Periods</strong></td>
<td>5</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td><strong>Nodes</strong></td>
<td>37</td>
<td>88</td>
<td>153</td>
</tr>
<tr>
<td><strong>Headways</strong></td>
<td>9-19, 11-17, 10-18, 8-20</td>
<td>6-10, 4-12</td>
<td>9-10, 4-12</td>
</tr>
<tr>
<td><strong>Syn. nodes</strong></td>
<td>35</td>
<td>271</td>
<td>1201</td>
</tr>
<tr>
<td><strong>Window times</strong></td>
<td>10-29, 5-16, 6-33, 10-14, 5-29</td>
<td>4-34, 4-18, 10-26, 1-16, 10-29, 2-22, 7-28, 4-15, 5-34</td>
<td>7-28, 7-18, 5-28, 4-19, 9-27, 1-21, 10-30, 2-19, 13-29, 8-15</td>
</tr>
</tbody>
</table>

For the small instance we got 14 efficient solutions with MOTS and 199 efficient solutions with SSPMO. In Figure 2 we can see that the distance to the ideal point (center of graph) in cost, synchronization and waiting time are very similar with both methods but the penalty is bigger with MOTS, however the difference is despicable.

Figure 2: Distance to ideal point small instance.

With the medium instance we got 19 efficient solutions with MOTS and 99 efficient solutions with SSPMO. In Figure 3 we can show that most of the objectives in both methods have a distance very similar to the ideal point (center of graph). However, the distance of the penalty obtained with SSPMO in this instance is bigger.

Figure 3: Distance to ideal point medium instance.

In the large instance we got 21 efficient solutions with MOTS and 99 efficient solutions with SSPMO. In Figure 4 we can see that both methods are in very similar distance in cost and synchronizations. However, the difference between them is penalty and time, for while MOTS is weak
in time, SSPMO is weak in penalty. In this case, both solutions could be attractive for the decision maker according to his preferences.

Figure 4: Distance to ideal point large instance.

In Table 3 we presented the summary of results that had been accomplished in each kind of instance with each metaheuristics, as well as the execution time.

<table>
<thead>
<tr>
<th>F.O.</th>
<th>Pequeñas</th>
<th>Medianas</th>
<th>Grandes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MOTS</td>
<td>SSPMO</td>
<td>MOTS</td>
</tr>
<tr>
<td>Cost</td>
<td>.51</td>
<td>.49</td>
<td>.50</td>
</tr>
<tr>
<td>Sincr.</td>
<td>.47</td>
<td>.53</td>
<td>.46</td>
</tr>
<tr>
<td>T. Trans.</td>
<td>.49</td>
<td>.51</td>
<td>.47</td>
</tr>
<tr>
<td>Penalidad</td>
<td>1</td>
<td>0</td>
<td>.10</td>
</tr>
<tr>
<td>T. Ej.</td>
<td>30</td>
<td>84420</td>
<td>1320</td>
</tr>
</tbody>
</table>

7 Ranking portfolios.

To establish the preference values we simulated a decision maker. We consider as the indifference threshold the 5% and for the preference threshold the 20% for all objectives. These values are taken with respect to the range of variation of the values of the objectives of the alternatives.

With respect to the weights in Table 4 the cost is considered as the main objective (a weight of 50% is assigned), the synchronizations and the transition between periods are equally important (20% for each one) and the less important is the transferring time (10%).

<table>
<thead>
<tr>
<th>Peso</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>0.50</td>
</tr>
<tr>
<td>Synchronization</td>
<td>0.20</td>
</tr>
<tr>
<td>Time</td>
<td>0.10</td>
</tr>
<tr>
<td>Transition</td>
<td>0.20</td>
</tr>
</tbody>
</table>

In Figure 5 we expose the comparisons between MOTS and SSPMO from the overrating relation. These results have been obtained applying Promethee I to the set of alternatives, conformed for the efficient solutions given for both methods in a particular instance. In each column we present
blocks of efficient solutions obtained with both methods, the superior block overrates the solutions of the inferior block.

Example, in the small instance the first block has 138 efficient solutions of SSPMO, these solutions overrate the three solutions of the second block, but these three solutions overrated the solutions of the third block, and so on.

From these results is possible to affirm that SSPMO gives solutions of better quality (with respect to the proximity of Pareto front) that MOTS. These comparisons provide a criteria to discard efficient solutions: if an efficient solution generated by one of both methods is outranked by at least one solution of the other method, then it will be discarded. Also can be used to choose a group of solutions that will be presented to the decision maker. For example, we can choose the first five solutions according to Promethee.

8 Conclusions

In this work it had been defined for the first time a mathematical model for urban bus planning that collects characteristics as: integrated frequency and timetabling, considering multiple periods with smooth transitions between periods with different demands and multiple objectives representing interests from all social actors involved. It also had been defined the limits and the scope of this model by the establishment of a set of assumptions that determined the validity in the application of the model in real cases.

We developed a support decision methodology to assist the decision maker in the first three phases of the decision making process helping him to structure the problem, to establish his preferences and to choose rationally those solutions that account to be acceptable trade offs among the objectives.

The experimental results made under a set of instances generated semirandomly (controlling certain characteristics as: instance size, density and demand) shows that SSPMO provides solutions of better quality that MOTS (closer to real Pareto front) but the execution time is significantly higher as the times needed by MOTS for solving the same instances.

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