A biobjective model for the integrated frequency-timetabling problem

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Abstract

The process of urban public transportation planning commonly include four basic activities usually executed sequentially: Network design, Timetabling, Vehicle scheduling and Crew scheduling. In this work we present an integrated model for the minimum frequency and the timetabling construction. The main scientific contribution of this paper is the development of an integrated MILP model to construct timetable by selecting frequencies in such a way that two objectives are optimized, like operational cost and synchronization. We use an exact method and two metaheuristics to solve the problem. Finally we show and analyzed some experimental results.

Keywords: frequency; timetabling; biobjective; integrated; buses.
1 Introduction

The process of urban planning transport includes commonly four basic activities, usually executed in sequence: Network design, Timetabling, Vehicle scheduling and Crew Scheduling. These activities according to Desaulniers & Hickman [6] are divided into subsystems: strategic, tactic and operational. The objective of the strategic subsystem is to maximize the service quality under budget restrictions, while the tactic subsystem is about the decisions offered to the public, these subsystems also focus on the quality of the service; finally the operational planning process is about how the operation should be made to give the propose service to a lower cost, clearly the objective of the operational planning is to minimize the total cost.

According to Desaulniers & Hickman [6] the problem of the minimum frequencies consist in select the frequencies that maximize the service of the passengers, this include restrictions of fleet size, capacity and policies headways for a minimum desired of frequencies. The minimum frequency problem, is a fundamental problem for the schedulers because at least they have two objectives in conflict: the companies wish to minimize the operation costs and the passengers desire a minimum travel time.

The construction process of a timetable consist in convert the desired frequencies into a timetable, the objective of timetable construction is to minimize the transfer times of the passengers, reflects the community needs of transportation and it is the base of the success for the next activities of the public transport process as noted by Borndorfer et al.[2]

The minimum frequency calculation and the timetable construction are the two problems that schedulers face not often. Frequency calculation and timetable construction are planned for the long term. If the timetable is not planned correctly the schedulers used to modify it based on their experience and this can cause implications for the vehicle scheduling and crew scheduling activities.

The importance of the timetables according Desaulniers & Hickman [6] is to determine the intervals between the vehicles for different periods of the day, the timetables indicate the times required for a user to transfer to one point to other. In general timetables are important for a company because they help to maintain under control salaries, and others aspects of the operation, and the timetables are the base for a controlled and orderly operation.

The main scientific contribution of this work is the development of an integrated (frequency and timetabling) bi-objective mathematical model to construct multiperiod urban bus timetables.

2 Problem Description

The planning process is divided into four phases: network design, timetabling, vehicle scheduling and crew scheduling [4]. The timetabling phase has two activities frequency determination and timetable construction, these activities are executed sequentially.

The problem addressed in this work is about to develop a mathematical model to calculate, in an integrated way, the minimum frequency and apply different criteria to create the timetable that give a better scheduling in public transport, giving benefits to social actors involved in the process,
like transport agencies; their goal is to offer to the population quality in the services with a low fare and maximize their profits, other social actor are the passengers who wants to minimize their travel and transfer times, more comfort trough their travel and a low fare. We have named this problem as FTT, and in was follows we will refers to the problem by this acronym.

Additionally, we are taking into account two objectives derived from the requirements of the social actors involved in the process, like synchronization, when two bus routes with departure times in the same period arrive to a specific node into a time window and the other objective we are considering is the cost of operation for the transport agency.

The challenge is to develop the planning for different periods with different demand. Recently Ibarra-Rojas & Ríos-Solís [9] have shown formally that the timetabling problem is NP-Hard, so another challenge in this work is to develop efficient algorithms for solving the FTT problem.

3 Related Work

Through the literature we can find that the timetabling problem have been tackled from different ways, one of them is the utilization of exact methods such as case Ceder [3] he creates a timetable with maximal synchronization. Also Eranki [7] proposes a model to create timetables with maximal synchronization using time windows, she uses a heuristic to solve the problem, but she did not consider more criteria.

There are other researches about timetables that use constraint programming [1] where the author presents a model considering different characteristics of the transport system (passenger requirements, budget constraints, level of service) and they solve it with constraint programming.

Exist some other researches of timetables where the authors use heuristics like GRASP, we can mention Mauttone & Urquhart [10] they develop an heuristic based on GRASP optimizing simultaneously different objectives of passengers and schedulers.

Trough the literature there are approaches, combining two phases of the urban transport process such as Szeto & Wu [11], they propose an integrated solution for the bus network design and frequency setting problems simultaneously using a genetic algorithm, which tackles the route network design problem is hybridized with a neighborhood search heuristic, which tackles the frequency setting problem; there are also approaches solving two phase sequentially, like the research made by Chakroborty [5], who combines the transit routing and scheduling phases using a genetic algorithm, in his approach he tries to minimize the transfer time and the waiting time; another research that combines several phases is the one proposed by Zhao & Zeng [12] they present a metaheuristic method for optimizing transit networks, including route network design, vehicle headway and timetable assignment, the goal is to identify a transit network that minimizes a passenger cost function; their metaheuristic combines simulated annealing, tabu and greedy search methods.

There are approaches about timetable that tackled the problem considering different criteria, visualize the problem like a multiobjective, approaches that combines (sequentially or integrated) different phases of the transport system, even exists investigations that talk about the smooth transition between periods, such as Ceder [4], he proposes two techniques to handle the smooth
transitions between periods with different demand; even headways with smooth transitions and headways with even average loads but there is no article in the literature reviewed that integrates simultaneously the minimum frequency problem and the timetabling problem considering these two objectives as we proposed in this paper.

4 Mathematical model

4.1 Assumptions.
Here we present the assumptions of the problem addressed in this work:

- Demand does not change significantly in each period and it is known in advanced.
- Average travel time from each route in each period is known.
- Periods length must be enough to allow the schedule the needed departures.
- The planning requirements must ensure the satisfaction of the demand during the planning period established.
- We are taking into account those departures within the same period for synchronization.

4.2 Mathematical model.

Sets
- $M$: Set of routes.
- $K$: Set of nodes.
- $V$: Set of periods.
- $B_{ij}^v$: Set of pairs of nodes where potentially synchronize the routes $i$ and $j$.
- $J(i)$: Set of routes which have common nodes with the route $i$.

Variables
- $X_{ip}^v = 1$ there is a trip in the route $i$ with departure time in the interval $(p \cdot H_{\text{min}_i}^v, p + 1 \cdot H_{\text{min}_i}^v + g)$ in the period $v$ y 0 otherwise.
- Where $p + 1 < |N^v| \ \text{and} \ \ 0 < g$ if $T_{\text{max}}^v - T_{\text{ini}}^v \ |N^v| \cdot H_{\text{min}_i}^v + g$
- $\alpha_{ip}^v \in (p \cdot H_{\text{min}_i}^v, p + 1 \cdot H_{\text{min}_i}^v + g)$ iff $X_{ip}^v = 1$, $\alpha_{ip}^v = 0$ iff $X_{ip}^v = 0$
- $Y_{ijkupq}^v = 1$ if the bus of the route $i$ with departure time in the interval $p$ and the bus of the route $j$ with departure time in the interval $q$ in the period $v$, arrive to the segment $k - u$ within the window time and 0 otherwise.

Parameters
- $G_v^v$: Number of trips in the period if we use a frequency equal to $H_{\text{max}}^v$.
- $P_{\text{max}_i}^v$: Maximum load of passengers in the route $i$ in the period $v$.
- $P_{\text{max}_d}^i$: Maximum load of passengers on bord in the day in the route $i$.
- $d_{\text{i}}^v$: Desired occupancy of the bus in the route $i$ in the period $v$.
- $P_{\text{as}_i}^v$: Total passengers/km in the route $i$ in the period $v$.
- $L_i^v$: Length of the route $i$.
- $\text{cap}_i^v$: Bus capacity of the route $i$ in the period $v$.
- $l_k^v$: Length of the segment $k$.
- $\beta_i^v$: Percentage allowed of the route $i$ of exceed the load in the period $v$. 
$H_{\text{max}}^v$: Minimum headway of the route $i$ in the period $v$.

$H_{\text{min}}^v$: Maximum headway of the route $i$ in the period $v$.

$T^v$: Planning period; $[T_{\text{ini}}^v, T_{\text{fin}}^v]$.

$T_{\text{ini}}^v$: Beginning time of the planning period $v$.

$T_{\text{fin}}^v$: Ending time of the planning period $v$.

$\gamma_i^v$: Desired time before the end of the period $T^v$ for the last departure of the route $i$ in the period $v$.

$W_{\text{max}}^v$: Maximum window time for the route $i$ in the period $v$.

$W_{\text{min}}^v$: Minimum window time for the route $i$ in the period $v$.

$t_{ik}^v$: Travel time from the origin point of the route $i$ to the segment $k$ in the period $v$.

$\delta_{ijk}^v$: Minimum time the passenger needs to change from segment $k$ of the route $i$ to the segment $u$ of the route $j$ in the period $v$.

$P_{\text{max}}^v$: Maximum load average of passengers on bord of route $i$ in the period $v$.

$MC^v$: Method applied to determine the frequency in the period $v$.

$f_{\text{mr}}^v$: Minimum frequency required to satisfy the demand of the route $i$ in the period $v$.

$f_{\text{mr}}^v = \frac{\sum_{p \in N^v} X_{ip}^v}{d_i^v}$.

$FixedCost_i^v$: Fixed cost for the route $i$ in the period $v$.

$VariableCost_i^v$: Variable cost for the route $i$ in the period $v$.

$P_k^v$: Average of passengers on bord in the segment $k$ in the period $v$.

$s_{jk}^v$: Holding time of the route $j$ in the segment $k$ during the period $v$.

$tc_i^v = \lceil \frac{H_{\text{max}}^v}{H_{\text{min}}^v} \rceil$

$tf_i^v = \lfloor \frac{H_{\text{max}}^v}{H_{\text{min}}^v} \rfloor$

$tm_i^v = \lceil -T^v_{\text{ini}} +(T^v_{\text{fin}} - \gamma_i^v) \rceil$

$mcp_i^v = \max\{p - tc_i^v, N^v\}$

$mfp_i^v = \max\{p - tf_i^v, N^v\}$

$msp_i^v = \max\{p - 1, 1\}$

$mu_i^v = \min\{tm_i^v + 1, N^v\}$

$FreMin_i^v = \max\{|G^v|, \sum_{p \in N^v} X_{ip}^v, f_{\text{mr}}^v\}$

\[
\min \sum_{i \in M} \sum_{v \in V} (\sum_{p \in N^v} X_{ip}^v \cdot FixedCost_i^v + VariableCost_i^v \cdot L_i \cdot \sum_{p \in N^v} X_{ip}^v) \tag{1}
\]

\[
\max \sum_{i \in M} \sum_{j \in J(i)} \sum_{(k,u) \in B_{ij}^v} \sum_{v \in V} \sum_{p \in N^v} \sum_{q \in N^v} Y_{ijkupq}^v \tag{2}
\]

s.t.

\[
MC^v == 1 \Rightarrow \sum_{p \in N^v} X_{ip}^v \geq \frac{P_{\text{max}}^d_i}{d_i^v}; \quad \forall v \in V; \forall i \in M \tag{3}
\]
\[ MC^v = 2 \Rightarrow \sum_{p \in N^v} X_{ip}^v \geq \frac{P_{\max_i}^v}{d_i^v}; \quad v \in V; i \in M \] (4)

\[ MC^v = 3 \Rightarrow \sum_{p \in N^v} X_{ip}^v \geq \frac{P_{\max_i}^v}{d_i^v \cdot L_i} \& \sum_{p \in N^v} X_{ip}^v \geq \frac{P_{\max_i}^v}{\text{cap}_i^v}; \quad v \in V; i \in M; \] (5)

\[ MC^v = 4 \Rightarrow \sum_{p \in N^v} X_{ip}^v \geq \frac{P_{\max_i}^v}{d_i^v \cdot L_i} \& \sum_{p \in N^v} X_{ip}^v \geq \frac{P_{\max_i}^v}{\text{cap}_i^v} \& \sum_{k \in I^v} l_k \leq \beta_i^v \cdot L_i; \quad v \in V; i \in M; \]
\[ I^v = \{ k | \frac{P_k^v}{f_{mfr}^v} \geq d_i^v \} \] (6)

\[ \alpha_{ip}^v \leq X_{ip}^v \cdot (T_{ini}^v + H_{\min_i}^v \cdot p) \quad \alpha_{ip}^v \geq X_{ip}^v \cdot (T_{ini}^v + (p - 1) \cdot H_{\min_i}^v + 1) \quad \forall v \in V, \forall i \in M, \forall p \in N^v \] (7)

\[ \sum_{p \in N^v} X_{ip}^v \geq \text{FreMin}_i^v; \quad v \in V; i \in M \] (8)

\[ \alpha_{i(tc_i^v)}^v \leq \sum_{c=1}^{t_i^v} X_{ic}^v \cdot M + (T_{ini}^v + H_{\max_i}^v), \sum_{c=1}^{(tc_i^v)} X_{ic}^v \geq 1 \quad \forall v \in V, \forall i \in M \] (9)

\[ \alpha_{ip}^v - \alpha_{i,(mcp_i^v)}^v \leq T_{fin}^v \cdot \sum_{c=mfp_i^v}^{msp_i^v} X_{ic}^v + X_{i,(mcp_i^v)}^v \cdot H_{\max_i}^v \]

\[ \alpha_{ip}^v - \alpha_{i,(msp_i^v)}^v \leq T_{fin}^v \cdot (1 - X_{i,(msp_i^v)}) + X_{ip}^v \cdot H_{\max_i}^v \]
\[-(T_{fin}^v) \cdot (1 - X_{i, (msp_i^v)}^v) - T_{fin}^v \cdot (1 - X_{ip}^v) \leq \alpha_{ip}^v - \alpha_{i, (msp_i^v)}^v - H_{min_i} \cdot X_{ip}^v\]

\[\sum_{c = mcp_i^v}^{msp_i^v} X_{ic}^v \geq 1 - (1 - X_{ip}^v) \cdot (N_i^v + 1)\]

\[\forall v \in V, \forall i \in M\] (10)

\[\alpha_{i, (tm_i^v)}^v \geq -M \cdot \sum_{c = ma_i^v}^{N_i^v} X_{ic}^v + (T_{fin}^v - \gamma_i^v)\]

\[\sum_{c = tm_i^v}^{N_i^v} X_{ic}^v \geq 1\]

\[\forall v \in V, \forall i \in M, \forall p, l \in N_i^v\] (11)

\[-1 \cdot (\alpha_{ip}^v + t_{ik}^v + \delta_{ijk}^v) + (\alpha_{jq}^v + t_{ju}^v + s_{jk}^v) \geq W_{min_i}^v - M \cdot (1 - Y_{ijkupq}^v)\]

\[-1 \cdot (\alpha_{ip}^v + t_{ik}^v + \delta_{ijk}^v) + (\alpha_{jq}^v + t_{ju}^v + s_{jk}^v) \leq W_{max_i}^v + M \cdot (1 - Y_{ijkupq}^v)\]

\[\alpha_{ip}^v + \alpha_{jq}^v \geq 2 \cdot Y_{ijkupq}^v\]

\[v \in V; i \in M; (k, u) \in B_{ij}^v; j \in J(i); p, q \in N_i^v,\] (12)

The model consists of 2 objective functions, the first objective function (1) minimize the total cost. We are considering a fixed and a variable cost which it is affected by the long of the route and the quantity of departures made by the route in the period. The second function (2) is to maximize the number of synchronizations between two bus routes with departure time in the same period.

These objective functions are subjected to constraints of frequency (3-6) which indicate the minimum quantity of units that a route needs to satisfy the demand. The method I (3) represents the maximum load point during the day, the method II (4) is based on the maximum load point during the period, method III (5) warranties the segment with maximum load will not present overcrowding and the method IV (6) sets a service level restricting a percentage of the total length of the route with overcrowding.

With constraint (7) we say if there is not a departure in the period v for the route i in the segment p then we do not assign a departure time. In constraint (8) the quantity of departures must be the maximum between the number of departures determined by the maximum headway, the minimum frequency and the frequency got it whit a method, which satisfies the maximum load point, in this way we are warranting accomplish of the demand.
Constraint (9) indicates that the first departure time must be less or equal to the maximum headway. Constraint (10) is for the consecutive departures, here indicates that the departure time must be between the minimum and maximum headway. For the last departure (11) we say the departure time must be between the end of the period and desired time.

The constraint (12) represents the synchronization which indicates when two bus of different routes arrives to a synchronization node between a window time and taking into account the transfer times, the permanence time in a node and the travel time, then there is a synchronization.

5 Computational experiments

We generated randomly 25 instances. The generator was coded in OPL (Optimization Programming Language), and we classified the instances in small, medium and large according to the quantity of routes, periods, segments and the density of synchronization nodes. See Table 1.

The denomination of small instances starts with P, the denomination of the medium instances starts with M and the denomination of large instances starts with L, then each name of the instance is followed by a P, M or D, these indicate low, medium or high density of synchronization nodes, the first number indicates the quantity of periods and the last numbers means the quantity of segments. For example, the instance PP-5-37 is a small instance with low density, 5 periods and 37 segments.

<table>
<thead>
<tr>
<th></th>
<th>Small</th>
<th>Medium</th>
<th>Large</th>
</tr>
</thead>
<tbody>
<tr>
<td>Routes</td>
<td>2-4</td>
<td>5-8</td>
<td>8</td>
</tr>
<tr>
<td>Periods</td>
<td>3-5</td>
<td>8-10</td>
<td>8-10</td>
</tr>
<tr>
<td>Nodes</td>
<td>10-18</td>
<td>19-23</td>
<td>35-50</td>
</tr>
<tr>
<td>Syn. Density</td>
<td>1%-2%</td>
<td>2%-4%</td>
<td>4%-7%</td>
</tr>
</tbody>
</table>

Table 1: Size of instances.

We implemented a metaheuristic to solve the problem, MOTS (Multiobjective Tabu Search) [8] developed in OPL. Also we solved the problem using CPLEX, we limited the time to 3600 seconds and a gap of 0.05.

After the experiments and according to the gap we obtained, we have classified the instances into easy, difficult and very difficult. The gap of easy instances in most of the cases is around 5%, the difficult instances have a reasonable gap around 10% but in some cases we got a gap around 5%, the very difficult instances gave a gap between 60% and 80%, this is the reason the instance PP-3-37 is classified as a very difficult instance. See Table 2.

We performed a test of sensitivity analysis of the amplitude of headway (minimum and maximum), we noticed that a margin of amplitude of 20 is sensible to the change. For example, when we decreased the amplitude of the headway to 8 in the PP-3-37 considered as a very difficult instance, we discovered the solution time decreases and we obtained optimal solutions with a gap of 5%.

We took several instances and we increased and decreased the amplitude of the headway; when we increased the amplitude the instance became into difficult instance, but we the amplitude de-
creases then it became into an easy instance; but the instances with high synchronization density are always difficult to solve, no matter the headway.

After this, we compared the results of MOTS with the results of CPLEX method, we just used the dominance test and in all cases not even one solution of MOTS dominated the solution obtained with the CPLEX method. In the small and medium instances the results are very similar to the CPLEX solutions, but in large instances the results gotten are not satisfactory.

With the very difficult instances, the solutions are not comparable (no solution of either method dominates a solution of the other method) in most of the cases, with difficult instances CPLEX solutions dominates MOTS solutions and easy instances the solutions gotten with CPLEX dominates the MOTS solutions, but in this case MOTS has an acceptable performance.

6 Conclusions

We show that our mathematical model can solve efficiently small, medium and large instances with low, medium and high synchronization density for the limits imposed of 1 hour and a gap of 5%, in general performs well, however, we can notice those limits could be relax even more in such a way that the algorithm or solution method performs efficiently.

Given that, for large instances we did not get satisfactory solutions, we implemented one of the metaheuristics most popular: MOTS, and we proved that it gave comparable solutions with the CPLEX, but MOTS performs better in execution time than CPLEX. Also, in the experiments we identified 2 factors affecting the structural complexity of the instances: amplitude of headway and synchronization density.

We can conclude that, CPLEX is efficient in all kind of instances with simple structural complexity, that means an amplitude not too large between the minimum and maximum headway and a density of synchronization nodes not too high; however for the instances with a hard structural complexity (large amplitude and high density) the execution time of 10 hours results insufficient to get efficient solutions with the instances we tested. At higher amplitude of headway and density of synchronization will be harder to solve the instance.
In the future we would like to experiment with column generation, decomposition techniques and valid inequalities to improve our mathematical model. Also, we would like to develop metaheuristics better adapted to the structure of the problem. Besides, we will incorporate more objectives that give benefits to other social actors involved in the process of planning transport, and to incorporate the uncertain of the demand.

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